Examples of Representations

WDRP - Representation Theory of Finite Groups

1. Determine a representation of D_4 of dimension $2n$ for each $n \in \mathbb{N}$.

2. Determine the matrix representations for the elements of V_4 (the Klein four group) for the left regular representation. Are there any subspaces fixed by all elements? (Hint: Try looking at eigenvectors)

3. Recall $\mathbb{Z}_3$ is the cyclic group with three elements $\{0, 1, 2\}$.

   a) Determine the matrix representation of each element of $\mathbb{Z}_3$ for the left regular representation (ρ, V).

   b) Find the *real* eigenvalues of the matrix representation $\rho(1)$, as well as the associated eigenspace.

   c) You may have noticed there is only one eigenvector in part b), call it v. Find two other vectors u, w orthogonal to v so that v, u, w forms an orthonormal basis for V (One way would be to use the Gram-Schmidt algorithm).

   d) Write $\rho(1)$ as a block diagonal matrix with respect to this basis (ie. how does $\rho(1)$ act on the subspace spanned by $\{u, w\}$?).

   e) Using the same characteristic polynomial you found for $\rho(1)$, determine both the real *and* complex eigenvalues, as well as the associated eigenspace. Use this to diagonalize $\rho(1)$.

   f) Is $\rho(2)$ diagonal with respect to this complex eigenspace?

4. Determine the rotational symmetries of an Octahedron. How many are there? (Hint: Any rotation has an axis of rotation which stays fixed.) Then give a representation of D_4 using these symmetries.

5. Let $\alpha : G \times V \to V$ be a structure map defining an action of G on a vector space V. Show that if the function $\alpha(g, -) : V \to V$ is linear for each $g \in G$, then this action defines a group representation.

6. Consider the following representations of D_3. Are these representations Isomorphic?

   (a)
$$\rho_1(r) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \rho_1(s) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   (b)
$$\rho_2(r) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \rho_2(s) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$