Euler's Formula and Generalizations

WDRP - Polyhedra

This is a short section. Basically Euler's formula is $V + F - E = 2$. The proof uses the fact that trees have $n - 1$ edges where n is the number of vertices. Take a polyhedra, start at one vertex and, using the edges, connect it to all other vertices so that it makes a tree. Now take the dual of the polyhedra and with the edges corresponding to the remaining edges of the original polyhedra make a tree connecting vertices again. The number of vertices in the first tree is the number of vertices of the polyhedron. While the vertices in the dual tree is the number of faces of the original polyhedron, so $E_1 = V - 1$ and $E_2 = F - 1$ where E_1 and E_2 are the number of edges in the first and second tree, respectively. Adding these together gives $E = E_1 + E_2 = V + F - 2$, which is Euler's formula if you move some stuff around.

So why is Euler's formula important? It helps us characterize an **invariant**, namely the "holiness" of polyhedra. This is because Euler's formula generalizes to $V + F - E = 2 - 2T$ where T is the number of tunnels or holes in the polyhedra. This helps us classify polyhedra up to the number of holes, so we are essentially comparing polyhedra by the number of holes they have.