1b) $(\alpha_p)^D \simeq \alpha_p$ $\qquad K[\alpha_p] = K[x]/x^p$ ← Has basis $\{1, x, \dots, x^{p-1}\}$

$$\left(K[x]/x^p\right)^D = \{\xi_0, \xi_1, \dots, \xi_{p-1}\}$$

$$\xi_i(x^j) = \delta_{ij}$$

$\Delta(x) = x \otimes x$ $\qquad \Delta^*(\xi_0, \xi_1)(x) = (\xi_0, \xi_1)(1 \otimes x + \cancel{x \otimes 1})$

$= 1$

$$\Delta^*(\xi_i, \xi_j)(a) = \sum \xi_i(a_1)\,\xi_j(a_2)$$

$$\binom{i+j}{i} = \frac{(i+j)!}{i!\,j!}$$

$\varphi : A^D \longrightarrow A$ $\qquad m(x^i, x^j) = x^{i+j}$

$\xi_i \longmapsto x^i$

$$\Delta^*(\xi_i, \xi_j)(x^{i+j}) = (\xi_i, \xi_j)\,\Delta(x^{i+j})$$

$$= (\xi_i, \xi_j) \sum \binom{i+j}{k} x^k \otimes x^{i+j-k}$$

$$= \sum \binom{i+j}{k} \xi_i(x^k) \cdot \xi_j(x^{i+j-k})$$

$$= \frac{(i+j)!}{i!\,j!}(i!\,,\,j!) = (i+j)! = (i+j)!\,\xi_{i+j}(x^{i+j})$$

$\varphi(\xi_i) = \varphi\left(\frac{i!}{i!}\xi_i\right) = \frac{1}{i!}(\varphi(i!\,\xi_i)) = \frac{1}{i!} x^i$

Choose basis for $\left(K[x]/x^p\right) = \{0!\,\xi_0, 1!\,\xi_1, \dots, (p-1)!\,\xi_{p-1}\}$

$(\varphi \otimes \varphi \circ m^*(k!\,\xi_k))($

defined by being 1 on

WTS

$m^*(k!\,\xi_k) = \sum \xi_{k-\ell} \otimes \xi_\ell$

$k!(\xi_k \circ m($

$\Delta(x^k) = \sum \binom{k}{\ell} x^{k-\ell} \otimes x^\ell$

$= \sum \frac{k!}{\ell!\,(k-\ell)!} x^{k-\ell} \otimes x^\ell$

$= k! \sum \underbrace{\frac{x^{k-\ell}}{(k-\ell)!}}_{\xi_{k-\ell}} \otimes \frac{x^\ell}{\ell!}$

$$\begin{tikzcd} A \arrow[r, "\Delta"] & A \otimes A \\ A^D \arrow[u, "\varphi"] \arrow[r, "m^*"'] & A^D \otimes A^D \arrow[u, "\varphi \otimes \varphi"'] \end{tikzcd}$$

$\varphi \otimes \varphi\, m^*(k!\,\xi_k)$

$\Delta(\varphi(k!\,\xi_k)) = \Delta(x^k)$

4) Let G_α be a family of representable functors, and $G = \varprojlim G_\alpha$. Since the G_α are rep'able $\exists A_\alpha \; \forall \alpha$ s.t. $G_\alpha(R) \cong \mathrm{Hom}(A_\alpha, R)$. Then let $A = \varinjlim A_\alpha$.

WTS that A represents G. To do this, need to show each elt in $G(R)$ corresponds to a map in $\mathrm{Hom}(A, R)$.

Let $\overline{\varphi} \in G(R)$ which is an equivalence class of elts

$$(x,y)^k = \left(kx,\ ky + \left(\sum_{i=2}^{p} (ix)^p - [(i-1)x]^p - x^p\right)/p\right)$$

$$(x,y)^p = \left(\overset{0}{\cancel{px}},\ \cancel{p}y + \left(\sum_{i=2} \quad . \quad . \quad . \quad \right)\right)$$

NTS $\displaystyle\sum_{i=2}^{p}\left((ix)^p - [(i-1)x]^p - x^p\right) / p \not\equiv 0 \pmod{p}$

$$= \sum_{i=2}^{p}\left(i^p - (i-1)^p - 1\right)x^p = \sum_{i=2}^{p}\left[i^p - (i-1)^p\right]x^p - \underbrace{\sum_{i=2}^{p} x^p}_{(p-1)x^p}$$

$$= \cancel{(2^p - (1)^p - 1) + (3^p - (2)^p - 1) + \ldots}$$
$$\cancel{+ ((p-1)^p - (p-2)^p - 1) +}$$

$$(\cancel{2^p} - 1) + (3^p - \cancel{2^p}) + \ldots + (p^p - \cancel{(p-1)^p})$$

$$(p^p - 1)$$

$$\left(p^p - 1 - (p-1)\right) = p^p - \cancel{1} - p + \cancel{1} = p^p - p$$

But divide by p

$p^{p-1} - 1$ which mod p is not 0

$\mathbb{G}_m \longrightarrow \mathbb{G}_a \iff K[x] \longrightarrow K[t, 1/t]$

$$x \longmapsto \sum_{i \in \mathbb{Z}} \lambda_i t^i$$

$\varphi(-x)$

$$-\sum \lambda_i t^i = \varphi(S(x)) = S(\varphi(x)) = S\left(\sum \lambda_i t^i\right)$$
$$= \sum \lambda_i S(t)^i$$
$$= \sum \lambda_i t^{-i}$$

So

$$\cancel{0 \stackrel{?}{=} \varphi(x-x) = \varphi(x) + \varphi(-x) = \sum \lambda_i t^{-i} + \sum \lambda_i t^i}$$
$$\cancel{= \sum \lambda_{-i} t^i + \lambda_i t^i}$$
$$\cancel{= \sum (\lambda_{-i} + \lambda_i) t^i}$$

$$\cancel{\varepsilon\left(-\sum \lambda_i t^i\right) = \varepsilon\left(\lambda_i t^{-i}\right)}$$

$$\cancel{-1 \sum \lambda_i = \sum \lambda_i}$$

$$\Delta\left(\sum \lambda_i t^i\right) = \Delta(\varphi(x)) = \varphi \otimes \varphi(\Delta(x))$$
$$= \varphi \otimes \varphi(1 \otimes x + x \otimes 1)$$
$$\sum \lambda_i \lambda_j t^i \otimes t^j \qquad = 1 \otimes \sum \lambda_i t^i + \sum \lambda_i t^i \otimes 1$$

~~only time~~ = is when

$\cancel{i=j=0}$ $\qquad \cancel{i=j=0:\ \lambda_i \lambda_j 1 \otimes 1} \qquad \cancel{\lambda_i 1 \otimes 1 + \lambda_j 1 \otimes 1}$

i $\qquad \lambda_0 = 0 \quad \to$ so each $\lambda_i = 0$ $\qquad \cancel{(\lambda_i + \lambda_j) 1 \otimes 1}$

$\cancel{\lambda_i \lambda_j}$

$$\begin{array}{ll} i=0 & \boxed{\begin{array}{l}\sum \lambda_0 \lambda_j\, 1 \otimes t^j \\ \sum \lambda_i \lambda_0\, t^i \otimes 1\end{array}} \\ j=0 & \end{array} \qquad \left.\begin{array}{l} 1 \otimes \sum \lambda_i t^i = \sum \lambda_j\, t^j \otimes 1 \\ \sum \lambda_i t^i \otimes 1 \end{array}\right\}$$

$\cancel{\lambda_0 \lambda_j = \lambda_j}$ $\qquad \lambda_i \lambda_0 = \lambda_i \qquad i=j>0 \quad \lambda_i^2 = 0$

$\cancel{\lambda_0 = 1}$ $\qquad i \neq j > 0 \quad \lambda_i \lambda_j = 0$

b) $\mathbb{G}_a \longrightarrow \mathbb{G}_m \iff K[x, x^{-1}] \longrightarrow K[t]$

$$x \longmapsto \sum \lambda_i t^i$$
$$x^{-1} \longmapsto \left(\sum \lambda_i t^i\right)^{-1}$$

$$\underline{1} = \varphi(1) = \varphi(x \cdot x^{-1}) = \varphi(x) \cdot \varphi(x^{-1}) = \left(\sum \lambda_i t^i\right) \varphi(x^{-1})$$

$$\Rightarrow \varphi(x^{-1}) = \varphi(x)^{-1}$$

$$\sum \lambda_i (-t)^i = S\left(\sum \lambda_i t^i\right) = S(\varphi(x)) = \varphi(S(x)) = \varphi(x^{-1}) = \left(\sum \lambda_i t^i\right)^{-1}$$

$$\left.\begin{aligned} 1 &= \left(\sum \lambda_i t^i\right)\left(\sum \lambda_i (-1)^i t^i\right) \\ &= \sum_{0 \le i,j \le k} \lambda_i \lambda_j (-1)^i t^{i+j} \end{aligned}\right\} \quad \begin{aligned} &i = j = 0 \quad \lambda_0 \lambda_0 = 1 \\ &i, j > 0 \quad \lambda_i \lambda_j = 0 \end{aligned}$$

If K is reduced $\lambda_i \lambda_j = 0$ is only possible if $\lambda_i, \lambda_j = 0$ when $i = j$ we need a nilpotent elt of order 2.

So only map is tirivial.

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c) Consider the map ~~$\varphi^* : \mathbb{G}_m \longrightarrow \mathbb{G}_a$ so~~ $\varphi : K[x] \longrightarrow K[t, 1/t]$ defined as $x \longmapsto 1 - by$ where $b \ne 0 \in K$ s.t.

$$x^{-1} \longmapsto 1 + by \qquad b^2 = 0$$

Then $\varphi(x \cdot x^{-1}) = \varphi(x)\varphi(x^{-1}) = (1 - by)(1 + by) = 1 + b^2 y^2$
$= 1$

5b) $\mu_n = \{ r \in R : r^n = 1 \}$ $\quad A = K[x]/\langle x^n - 1 \rangle$

$\cancel{r^k \cdot r^s = r^{k+s}}$ $\quad g : x \mapsto r$

$(g \cdot f) \circ \Delta(x) = r \cdot s$ $\quad \Delta(x) = x \otimes x$

$g : x \mapsto r$ $\quad g(x) \cdot f(x) = r \cdot s$

$f : x \mapsto s$

$x = (\varepsilon, id) \circ \Delta(x) = \varepsilon(x) \cdot x$

$\varepsilon(x) = 1$

$1 = \varepsilon(x) = (S, id) \circ \Delta(x) = S(x) x$

$S(x) = x^{n-1}$

5c) $\alpha_p = \mathrm{Ker} \hookrightarrow \overset{A}{G_a} \longrightarrow \overset{B}{G_a}$

Since α_p is the kernel of the pth power map we have $\mathrm{Ker}(\alpha_p)$ is represented by $A \otimes_B K \cong A/I_B A$ where I_B is $\mathrm{Ker}(\varepsilon) = \{ x p(x) \mid \forall p(x) \in K[x] \} = (x) \subseteq B$, so because $\varphi : B \to A$ has $x \mapsto y^p$ $\quad I_B A = (y^p) A = (y^p)$

So α_p is rep'd by $K[t]/(t^p)$.

Moreover as a subgroup it inherit's the hopf structure from ~~G_a~~ $K[x]$ as the coord. alg of G_a.

$\Delta(x) = x \otimes 1 + 1 \otimes x$

$S(x) = -x$

$\varepsilon(x) = 0$

5a) $\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} x_{11}y_{11} + x_{12}y_{21} & x_{11}y_{12} + x_{12}y_{22} \\ & \end{pmatrix}$

$X_{11} \longmapsto X_{11} \otimes X_{11} + X_{12} X_{21} \qquad X_{12} \longmapsto X_{11} \otimes X_{12} + X_{12} \otimes X_{22}$

$X_{ij} \underset{\Delta}{\longmapsto} \sum_{k=1}^{n} X_{ik} \otimes X_{kj} \qquad \varepsilon(x_{ij}) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$x_{ij} = (\varepsilon, id)\,\Delta(x_{ij}) = \sum_{k=1}^{n} \varepsilon(x_{ik}) \otimes x_{kj}$

$\varepsilon(X_{ij}) = (S, id) \circ \Delta(X_{ij}) = \sum_{k=1}^{n} S(X_{ik}) \otimes X_{kj}$

$1 = \varepsilon(X_{11}) = S(X_{11})X_{11} + S(X_{12})X_{21}$

$0 = \varepsilon(X_{12}) = S(X_{11})X_{12} + S(X_{12})X_{22}$

$-S(X_{11})X_{12} = S(X_{12})X_{22}$

$\begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} \longmapsto \frac{1}{\det}\begin{pmatrix} X_{22} & -X_{12} \\ -X_{21} & X_{11} \end{pmatrix}$

$X_{22} = S(X_{11})X_{11}X_{22} + S(X_{12})X_{22}X_{21}$

$X_{22} = S(X_{11})X_{11}X_{22} - S(X_{11})X_{12}X_{21}$

$X_{22} = S(X_{11})(\det)$

$S(X_{11}) = \frac{1}{\det} X_{22}$ ← Cramer's Rule.

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6) $\Delta(x_{ij})$ because matrix coord fnc.

$\varepsilon(x_{ij})$ eval should be identity matrix.

2) Let E be a rep'ble functor by A and F any functor.

$\Rightarrow$) Let $\Phi : E \to F$ be a natural trans. consider the diagram

let $g : A \to R$ and R a k-alg.

$$\begin{tikzcd} E(A) \arrow[r, "E(g)"'] \arrow[d, "\Phi_A"'] & E(R) \arrow[d, "\Phi_R"] \\ F(A) \arrow[r, "F(g)"'] & F(R) \end{tikzcd}$$

Consider $id_A \in E(A)$. Then let $x = \Phi_A(id_A) \in F(A)$. By naturality

$\Phi_R(E(g)(id_A)) = F(g)(\Phi(id_A)) = g(x)$

$\Phi_R(g) = g(x) \quad \forall g \in E(R)$.

Thus, Φ depends on $x \in F(A)$.

$\Leftarrow$) Now, consider $x \in F(A)$, and define $\Phi_R(g) = F(g)(x)$ $\forall g \in E(R)$. Indeed Φ_R gives a map since $F(g) : F(A) \to F(R)$ So the image lies in $F(R)$. For naturality let S, R be k-alg w/ $h : S \to R$. Then for any $f \in E(S)$ $f : A \to S$

$$\begin{tikzcd} E(S) \arrow[r, "E(h)"] \arrow[d, "\Phi_S"'] & E(R) \arrow[d, "\Phi_R"] \\ F(S) \arrow[r, "F(h)"'] & F(R) \end{tikzcd}$$

$$\begin{aligned} \Phi_R(E(h)(f)) &= \Phi_R(h \circ f) \\ &= F(h \circ f)(x) \\ &= F(h) \circ F(f)(x) \end{aligned}$$

$$\begin{aligned} F(h)(\Phi_R(f)) &= F(h)(F(f)(x)) \\ &= F(h) \circ F(f)(x) \end{aligned}$$

So the diagram commutes,
which means Φ_R is a natural transformation.

1a) $\mathrm{Hom}(A, R\times S) \cong \mathrm{Hom}(A,R)\times \mathrm{Hom}(A,S)$

$\varphi(a) = (\pi_R\varphi(a), \pi_S\varphi(a))$

$\varphi \longmapsto (\pi_R\varphi, \pi_S\varphi)$

For reverse

$$\begin{tikzcd} S & A \arrow[l, "\varphi_2"'] \arrow[r, "\varphi_1"] \arrow[d, dashed, "\varphi"] & R \\ & R\times S \arrow[ul, "\pi_S"] \arrow[ur, "\pi_R"'] & \end{tikzcd}$$

By univ. prop

1b) Suppose $\exists$ a rep'ble functor F s.t. $F(R)$ has 2 elts for every k-alg R. Then for k-alg R, S,

$F(R\times S)$ has two elts. but we know from 1a)

$F(R)\times F(S)$, which has 4 elts, a contra.

1c) Suppose $A = k\times k$

$\to$ ~~If R is a k-alg w/ only 0 and 1 as idempotents~~

~~then $\mathrm{Hom}_{k\text{-alg}}(A,R)$ has two elts φ.~~

Note that A has basis elts. $e_1 = (1,0)$ and $e_2 = (0,1)$ which are idempotents satisfying

$e_1\cdot e_2 = 0 \qquad e_1 + e_2 = 1$

So f determined by

Moreover, for any $f: A \to R$, $1_R = f(1_A) = f(e_1+e_2) = f(e_1)+f(e_2)$

~~Suppose $f \in \mathrm{Hom}(A,R)$ then~~

Then because any $a\in A$ is uniquely written as $a_1e_1 + a_2e_2$ $a_1, a_2 \in k$ so $f(a) = f(a_1e_1) + f(a_2e_2) = a_1 f(e_1) + a_2 f(e_2)$

So f uniquely determined by $f(e_1)$ and $f(e_2)$

$\to$ If R only has 0, 1 as idem. then the only f's are $f: \begin{cases} e_1 \mapsto 0 \\ e_2 \mapsto 1 \end{cases}$ & g

$\to$ Suppose R has another idem. e' than 1, so f from before work

but so does $g: \begin{cases} e_1 \mapsto e' \\ e_2 \mapsto 1-e' \end{cases}$ & g_2 so $(1-e')$ is also an idempotent

which gives more than 2.

$$\begin{tikzcd} E(A) \arrow[r, "E(g)"] \arrow[d, "\Phi_A"'] & E(R) \ni g \arrow[d, "\Phi_R"] \\ x \in F(A) \arrow[r, "F(g)"'] & F(R) \end{tikzcd}$$

$\Phi_R(g) = \cancel{g(x)}\ F(g)(x)$

$A \to S$

$h: S \to R$ $\qquad$ $g: A \to R$

$$\begin{tikzcd} E(S) \arrow[r] \arrow[d] & E(R) \\ F(S) \arrow[r] & {} \end{tikzcd}$$

~~$f: A \to A$~~ $\qquad$ $h: S \to R$

$g: A \to R$

$$\begin{tikzcd} \mathrm{id}_A \in E(A) \arrow[r, "E(g)"] \arrow[d, "\Phi_A"'] & E(R) \ni g \arrow[d, "\Phi_R"] \\ x \in F(A) \arrow[r, "F(g)"'] & F(R) \end{tikzcd}$$

$g: A \to R$

$f: A \to S$

$$\begin{tikzcd} E(S) \arrow[r, "E(h)"] \arrow[d, "\Phi_S"'] & E(R) \arrow[d, "\Phi_R"] \\ F(S) \arrow[r, "F(h)"'] & F(R) \end{tikzcd}$$

$F(h)(\Phi_S(f)) = F(h)\,F(f)(x)$

$\neq$

~~$F(g)(\Phi_A(f)) = \Phi_R(E(g)(f))$~~

~~$F(g)(\Phi_R(f)) = \Phi_R(f \circ g) = F(f \circ g)(x)$~~

$\Phi_R(E(h)(f))$

$F(g)(\Phi_A(\mathrm{id}_A)) = \Phi_R(E(g)(\mathrm{id}_A))$

$\qquad ||$

$= \Phi_R(g) = F(g)(x)$

$F(g)(F(\mathrm{id}_A)x)$

$F(g)(\mathrm{Id}_{F(A)}x)$

$F(g)\,x$

$\Phi_R(h \circ f)$

$= F(h \circ f)(x)$

$= F(h) \circ F(f)(x)$

$$\cancel{(x+1)^2-1} = \cancel{x^2+2x}$$

$$(x+1)^4-1 = x^4+4x^3+6x^2+4x\cancel{+1}$$

$$\binom{p-1}{n}$$

$$\frac{(p-1)!}{n!\,(p-1-n)!}$$

n even

$$\frac{(p-1)!}{(2k)!\,(p-1-2k)!} \qquad \frac{(2m)!}{(2k)!\,(2m-2k)!}$$

$$\frac{2m\cdot 2m-1\cdot\ldots\cdot\cancel{(2k)!}}{\cancel{(2k)!}\,(2(m-k))!}$$

$$\frac{2m\cdot 2m-1\cdot\ldots\cdot 2k+1}{(2m-2k)\cdot(2m-2k-1)\cdot\ldots\cdot(}$$

$$
\begin{array}{r|l}
 & x^{p-1} - x^{p-2} + x^{p-3} - \dots - x + 1 \\
x+1 & x^p \\
 & -(x^p + x^{p-1}) \\ \hline
 & -x^{p-1} \\
 & -(-x^{p-2} - x^{p-2}) \\ \hline
 & x^{p-2} \\ \hline
 & -x^2 \\
 & -(-x^2 - x) \\ \hline
 & x
\end{array}
$$

$2k+1 - (2k'+1)$

$2k+1-3$

$2k+1-2+1$

$2(k-1)+1$

$\Delta(x) = \Delta(t-1) = \Delta(t) - \Delta(1) = t \otimes t - 1 \otimes 1$

So $A = K[t]/(t-1)^p \cong K[x]/x^p$ where $x = t-1$.

Since char $K = p > 0$, $t^p - 1 = t^p - 1^p = (t-1)^p$

$\Rightarrow y = t^{p-1}$ so $A = K[t]/(t^p-1)$

Ker φ_p rep'd by $A = K[t, 1/t]/t^p - 1 = K[t, y]/(ty-1)(t^p-1)$

$G_m \xrightarrow{\varphi_p} G_m \qquad \text{Ker } \varphi_p = \{x : x^p = 1\}$

$\mathcal{N}_p = \operatorname{Ker} \varphi \hookrightarrow \mathbb{G}_m \xrightarrow{p} \mathbb{G}_m$

$\mathcal{N}_p = \operatorname{Spec}(K[t]/t^p - 1)$

So in char $K = p > 0$ $\quad t^p - 1 = t^p - 1^p = (t-1)^p$

So $A = K[t]/t^p - 1 = K[t]/(t-1)^p \overset{n}{\cong} K[x]/x^p$ by change of variable

But as $\operatorname{Ker} \varphi$, $A = K[t, 1/t]/(t^p - 1) = K[t, y]/(t^p - 1)(ty - 1)$

$\Rightarrow y = t^{p-1}$ so $A = K[t]/(t^p - 1)$

So same change of variable gives $A = K[x]/(x^p)$

but then $x = t - 1 \Rightarrow t = x + 1$, so $y = (x+1)^{p-1}$

But then $ty - 1 = (x+1)(x+1)^{p-1} - 1 = (x+1)^p - 1 = x^p + 1 - 1 = \underset{=0}{x^p}$

Num that works, In my head I thought it was x not x^p so num,

---

$\Delta_{\mathcal{N}_p}(x) = \Delta_M(t-1) = \Delta(t) - \Delta(1) = t \otimes t - 1 \otimes 1$

Since $t = x + 1$

$$\begin{aligned} t \otimes t - 1 \otimes 1 &= (x+1) \otimes (x+1) - 1 \otimes 1 \\ &= x \otimes x + x \otimes 1 + 1 \otimes x + \cancel{1 \otimes 1 - 1 \otimes 1} \\ &= x \otimes x + x \otimes 1 + 1 \otimes x \end{aligned}$$

$\varepsilon_p(x) = \varepsilon_m(t-1) = \varepsilon(t) - \varepsilon(1) = 1 - 1 = 0$ $\qquad x = \varepsilon(x)x + \varepsilon(x) + x$

$\Rightarrow \varepsilon(x) = 0$

$S_p(x) = S_m(t-1) = S(t) - S(1) = t^{p-1} - 1$

$0 = \varepsilon_p(x) = (S, id) \circ \Delta(x) = S(x)x + S(x) + x$

$G_m \xrightarrow{\varphi_p} G_m \qquad \ker \varphi_p = \{x \in R : x^p = 1\}$

So $\ker \varphi_p$ rep'd by $K[t,t^{-1}]/(t^p-1)$

but in char p $\quad t^p - 1 = t^p - 1^p = (t-1)^p$

So $A = K[t,t^{-1}]/(t-1)^p$

$x = t-1 \Rightarrow t = x+1 \qquad K[x+1, 1/x+1]/x^p$

$$\frac{1}{x+1} = x^{p-1} + (p-1)x^{p-2} + \dots + (p-1)x + 1$$

$$(x+1)(x^{p-1} + (p-1)x^{p-2} + \dots + (p-1)x + 1)$$

$$= x^p + (p-1)x^{p-1} + \dots + (p-1)x^2 + x$$

$$+ \qquad x^{p-1} + \dots +$$

$x = t-1 \qquad K[t,y]/(ty-1)(t^p-1) \Rightarrow y = t^{p-1}$

$\cancel{x\otimes}\ \Delta(x) = \Delta(t-1) = \Delta(t) - \Delta(1)$

$= t \otimes t - 1 \otimes 1$

$t = x+1$

$= (x+1) \otimes (x+1) - (1 \otimes 1)$

$= x \otimes x + 1 \otimes x + x \otimes 1 \cancel{+ 1 \otimes 1 - 1 \otimes 1}$

$\varepsilon(x) = 1$

$x = (\varepsilon, id) \cdot \Delta(x) = \varepsilon(x) \otimes x$

$x = (\varepsilon, id)(\Delta(x)) = \varepsilon(1)\, x + \varepsilon(x)$

$$t \otimes t - 1 \otimes t - t - 1 \otimes 1 \cancel{+ 1 \otimes t = (t-1) \otimes 1 - 1 \otimes 1}$$

$$\downarrow$$

$$t - 1 \otimes t$$

$$\downarrow$$

$$t - 1 \otimes t - 1$$

$$t \otimes t - 1 \otimes t - t \otimes 1 + 1 \otimes 1$$

$$\cancel{t \otimes t - 1 \otimes 1} + 1 \otimes t - 1 \otimes t + t \otimes 1 - t \otimes 1$$

$$(t-1) \otimes t \qquad 1 \otimes t - 1$$

$$(x+1)(x+1)^{p-1}$$

$$(x+1)^p - 1$$

$$x$$

$$(1 \otimes x + x \otimes 1)(1 \otimes x + x \otimes 1)$$

$$1 \otimes x^2 + x \otimes x + x \otimes x + x^2 \otimes 1$$

$$\begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}\begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} n^2 & nc \\ 0 & 0 \end{bmatrix}\begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} n^3 & n^2c \\ & \end{bmatrix}$$

g unip. $f = g-1$

$$h = \log(g) = f - f^2/2 + f^3/3 - \dots \pm \frac{f^{m-1}}{m-1}$$

$\exp(h) =$

When g is 1×1, $g = [1]$, so $f = [0]$ so $\exp(\log(g)) = 1+0 = 1 = g$

Then suppose for g $k \times k$, $\exp(\log(g)) = g$.

Suppose g is $(k+1)\times(k+1)$

Then split g into block so that $f = g-1$ is of the form $\begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}$ where n is a nilpotent $k\times k$ matrix and c is a $k \times 1$ column. In particular n corresp. to a unip. matrix g_n.

Moreover $f^k = \left(\begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}\right)^k = \begin{bmatrix} n^k & n^{k-1}c \\ 0 & 0 \end{bmatrix}$

So $\exp(\log(g)) = \sum_{i=0}^{m-1} \frac{\log(g)^i}{i!} = \sum_{i=1}^{m-1} \frac{1}{i!}\begin{bmatrix} n^i & n^{i-1}c \\ 0 & 0 \end{bmatrix} + I_{k+1}$

$$= \begin{bmatrix} \sum_{i=1}^{m-1} \frac{n^i}{i!} + I_k & \left(\sum_{i=1}^{m-1} \frac{n^{i-1}}{i!}\right)c \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \exp(\log(g_n)) & \left(\sum_{i=1}^{m-1} \frac{n^{i-1}}{i!}\right)c \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} g_n & c \\ 0 & 1 \end{bmatrix} = g.$$

$$h^K = \begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}^K = \begin{bmatrix} n^K & n^{K-1}c \\ 0 & 0 \end{bmatrix}$$

$\exp(\log(g))$

$$\sum_{i=0}^{m-1} \frac{(h)^i}{i!} = \begin{bmatrix} \sum_{i=0}^{m-1} n^i/i! & \left(\sum_{i=1}^{m-1} n^{i-1}/i!\right) c \\ 0 & 0 \end{bmatrix}$$

$n = g_n - 1$

$=$

$$g = \begin{bmatrix} g_n & c \\ 0 & 1 \end{bmatrix} \qquad f = g - 1 = \begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}$$

$\cancel{h_n = \log(g_n)}$

$$f^K = \begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix}^K = \begin{bmatrix} n^K & n^{K-1}c \\ 0 & 0 \end{bmatrix}$$

$\cancel{n = \log(g_n)}$

$$\sum_{i=1}^{m-1} \frac{(-1)^{i+1} f^i}{i} = \sum_{i=1}^{m-1} \frac{(-1)^{i+1}}{i} \begin{bmatrix} n^i & n^{i-1}c \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{m-1} \frac{(-1)^{i+1}}{i} n^i & \boxed{\left(\sum_{i=1}^{m-1} \frac{(-1)^{i+1}}{i} n^{i-1}\right) c}^{\,c'} \\ 0 & 0 \end{bmatrix}$$

$$\exp\left(\begin{bmatrix} \log(g_n) & c' \\ & \end{bmatrix}\right) \qquad \begin{bmatrix} g_n & c' \\ & \end{bmatrix}$$

$$h_n = \log(g_n) = n - \frac{n^2}{2} + \frac{n^3}{3} \cdots$$

$$n\left(1 - n/2 + \frac{n^2}{3} \cdots\right)$$

$K[x,y]/(x^2+y^2=1)$

$V = \langle\{x,y\}\rangle$ $\quad \Delta(x) = \overset{v_1}{x}\otimes \underset{11}{x} - \overset{v_2}{y}\otimes y_{21}$ $\quad a_{11} = x$

$\Delta(y) = \overset{v_2}{y}\otimes \underset{22}{x} + \overset{v_1}{x}\otimes y_{12}$

$a_{21} = -y$ $\quad a_{12} = y$ $\quad a_{22} = x$ $\quad \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

R comm k-alg $\quad G_a(R) \longrightarrow GL_n(R)$

$\varphi(rh) =$ matrix w/ coeff in R $\{$

Invertibility

$$\left(\sum_{k=0}^{m-1} \frac{(rh)^k}{k!}\right)\left(\sum_{i=0}^{m-1} \frac{(-1)^i (rh)^i}{i!}\right)$$

$$\sum_{i=0}^{m-1}\sum_{k=0}^{m-1} \frac{(-1)^i (rh)^{k+i}}{i!\,k!}$$

pairs $(i,k) \in \underline{m-1} \times \underline{m-1}$

	0	✓ 1	✓ 2	✓	.. m-1
0	(0,0)	(0,1)	(0,2)	(0,3)	
1	(1,0)	(1,1)	(1,2)		
2	(2,0)	(2,1)			
.	(3,0)				
.					
m-1					

$n = i+k$

$$\cancel{\sum_{k=0}^{2m-2} \frac{1}{n!}\binom{n}{k}(-1)^k (rh)^n}$$

$$\left(1 + rh + \frac{(rh)^2}{2!} + \dots + \frac{(rh)^{m-1}}{(m-1)!}\right)\left(1 - rh + \frac{(rh)^2}{2!} + \dots \pm \frac{(rh)^{m-1}}{(m-1)!}\right)$$

$(-1)^i$

$\exp(rh) \cdot \exp(-rh)$

$= \left(1 + rh + \frac{(rh)^2}{2!} + \frac{(rh)^3}{3!} + \ldots\right)\left(1 - rh + \frac{(rh)^2}{2!} - \frac{(rh)^3}{3!} + \ldots\right)$

$\begin{array}{cc} i & k \end{array}$

$(0, n) \quad (1, n-1) \quad (2, n-2) \ldots ($

$\downarrow$

$\frac{(-1)^0 (rh)^n}{0!\, n!} \quad \frac{(-1)^1 (rh)^n}{1!\,(n-1)!}$

$\sum_{i=0}^{n} \binom{n}{i} \frac{1}{n!} (-1)^i$

$$h^K = \begin{bmatrix} \log(g_n) & pC \\ & \end{bmatrix}^K \qquad p = \sum \frac{(-1)^{i+1}}{i} n^{i-1}$$

$$= \begin{bmatrix} h_n^K & h_n^{K-1}(pC) \\ & \end{bmatrix}$$

$$g \stackrel{?}{=} \exp(h) = \sum_{K=1}^{m-1} \begin{bmatrix} h_n^K & h_n^{K-1}(pC) \\ 0 & 0 \end{bmatrix} \frac{1}{K!} + 1$$

$$= \begin{bmatrix} \exp(h_n) & \sum_{K=1}^{m-1} \frac{h_n^{K-1}}{K!}(pC) \\ & \end{bmatrix}$$

$$\left(1 + \frac{h_n}{2!} + \frac{h_n^2}{3!} + \dots\right)\left(1 - \frac{n}{2} + \frac{n^2}{3} - \dots\right)$$

$$g_n = \exp(\log(g_n)) = 1 + h_n + \frac{h_n^2}{2!} + \frac{h_n^3}{3!} + \dots$$

$$n = g_n - 1 = h_n\left(1 + \frac{h_n}{2!} + \frac{h_n^2}{3!} + \dots\right) \qquad = 1$$

$$h_n = \log(g_n) = n - \frac{n^2}{2} + \frac{n^3}{3} - \dots$$

$$= n\left(1 - \frac{n}{2} + \frac{n^2}{3} - \dots\right)$$

$$\left(\sum_{i=1}^{m-1} \frac{(-1)^{i+1}}{i} n^{i-1}\right)\left(\sum_{k=1}^{m-1} \frac{n^{k-1}}{k!}\right)$$

$$\left(1 - \frac{n}{2} + \boxed{\frac{n^2}{3}} - \frac{n^3}{4} \cdots\right)\left(1 + \boxed{\frac{n}{2}} + \frac{n^2}{3!} + \cdots\right)$$

$$1\left(\Sigma \quad\right) - \frac{n}{2}\left(\Sigma \quad\right)$$

$\log(g_n) = n($

$$\cancel{\frac{n^2}{3} - \frac{n^2}{4} + \frac{n^2}{3} + \frac{n^2}{3!}}$$

$h_n = \log(g_n)$

$$g_n = \exp(\underbrace{\log(g_n)}_{h}) = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \cdots$$

$$n = g_n - 1 = h\underbrace{\left(1 + \frac{h}{2!} + \frac{h^2}{3!} + \cdots\right)}$$

$$\left(\sum_{i=1}^{m-1} \frac{(-1)^{i+1}}{i} n^{i-1}\right) \cdot \left(\quad\right)$$

$$\underbrace{\left(\frac{n^0}{1} + \frac{n}{2!} + \frac{n^2}{3!} + \cdots + \frac{n^{m-2}}{(m-1)!}\right)}_{n} C = C$$

$$g_n = \exp(n) = 1 + n + \frac{n^2}{2} + \overset{n^3/3!}{\cdots} + \frac{n^{m-1}}{(m-1)!}$$

$$g_n = 1 + n\left(1 + \frac{n}{2} + \frac{n^2}{3!} + \cdots + \frac{n^{m-2}}{(m-1)!}\right)$$

$$n = g_n - 1 = n\underbrace{\left(1 + \frac{n}{2} + \frac{n^2}{3!} + \cdots + \frac{n^{m-2}}{(m-1)!}\right)}_{= I_k}$$

$\Rightarrow$

$$1 + \cancel{\log(g_n) + \frac{\log(g_n)^2}{2!}} \cdots$$

n

$$1 + \begin{bmatrix} n & c \\ 0 & 0 \end{bmatrix} + \frac{\begin{bmatrix} n^2 & nc \\ 0 & 0 \end{bmatrix}}{2!}$$

$1 + n + n^2$ $\quad + C$

$$C + \frac{nc}{2!} + \frac{n^2 c}{3!}$$

$$\left(1 + \frac{n}{2!} + \frac{n^2}{3!} + \cdots\right) C$$

⑤ $(c_{ij}) = (a_{ij})(b_{ij})$

$c_{ij} = \sum a_{ik} b_{kj}$ $\quad$ $\underline{j-i<r}$

$a_{ik} = 0 \quad k-i<r$

$b_{kj} = 0 \quad j-k<r$

If $k<j \quad k-i<j-i<r \Rightarrow a_{ik}=0$

if $k \geq j \Rightarrow -k \leq j \Rightarrow j-k \leq j-j = 0 < r \Rightarrow b_{kj}=0$

So $c_{ij}=0$

~~if k j~~

So H_r a subgroup

Pick smallest r s.t. $G \cong H \leq H_r$

Then $\varphi : H_r \to G_a$ restricts to a hom on H and therefore G.

$\mathrm{Nat}(h_A, F) = F(A)$

$\mathrm{Nat}(h_A, h_B) = h_B(A) = \mathrm{Hom}(B, A)$

$A = kG$

$g \mapsto g \otimes g$

$\cancel{\mathrm{Mo}}$

$\mathcal{F} : \mathrm{mod}\, A \longrightarrow \mathrm{Vec}_k$

$\underline{\mathrm{End}(\mathcal{F})} = \mathrm{Nat}(\mathcal{F}, \mathcal{F}) \cong h_A(A)$

$\mathrm{Hom}_A(A, M) \cong \mathcal{F}(M) \qquad \mathcal{F} \cong h_A$

$x \mapsto \cancel{1 \otimes x + x \otimes 1}$

$x \otimes 1 + 1 \otimes x$

$\mathrm{Hom}(k, M) = M^G$

$x \cdot (m \otimes n)$

$= xm \otimes n + m \otimes xn$

4. (12 pts) Find the absolute (*i.e.* global) minimum and maximum values of the function $z = f(x,y) = 5 + 2x^2 + y^2 + 8y$ on the region $D = \{(x,y) : y \geq 0, x^2 + y^2 \leq 25\}$.

(*For full credit: You must very clearly show that you found and analyzed the one variable functions above each boundary curve and that you checked all appropriate points and endpoints*)

global max: $z =$ ________

global min: $z =$ ________

You may use this page for scratch-work or extra room.
All work on this page will be ignored unless you write and circle "see scratch page" on the problem and you label your work.

$$R \otimes_{\mathbb{Z}} R \to R$$

$$q_{ij} = q_{ji}^{-1} \in K^{\times}$$

$$\cancel{[x, y]_q}$$

$$[x_i, x_j]_q = x_i x_j - q_{ij} x_j x_i$$

Math 126 DA, DB, DC Final - Kane 110

Sit "every-other" seat. To help with this, please leave an odd number of seats between you and the person next to you.

Turn off and put away your cell phone. Allowed items:

a. TI-30X IIS calculator

b. 8.5 x 11 in. sheet of notes

Raise your hand if you have a question. *We can clarify the wording of a question and we can comment on the form of the final answer, but we can't comment on your work.*

If you need the restroom, close your exam and put your phone on your exam and let the TA know.

Final grades will be on Canvas by Friday at the end of finals week. Dr. Loveless will send a message out the moment grades are public.

Put instructor first names only on the cover:

Instructor Name: **Andy**

Quiz Sections:

Spencer	DA Tues	1230
	DB Tues	130
Jay	DC Tues	1230

$M \otimes N$ is an $A \otimes A$ module

$\otimes \rightsquigarrow \otimes_k$

$\overline{\mathcal{F}} : \mathcal{C} \boxtimes \mathcal{C} \to \mathcal{C} \xrightarrow{\mathcal{F}} \mathrm{vec}_k$

$\Delta : A \to A \otimes A$

$\varepsilon : A \to k$

$S : A \to A$

$S^2 = \mathrm{id}?$

$\otimes : \mathcal{C} \boxtimes \mathcal{C} \to \mathcal{C}$

$\mathcal{F} : \mathcal{C} \to \mathrm{vec}_k$

$\mathrm{End}(\mathcal{F})$

$$\begin{tikzcd}
\mathrm{mod}\,A \boxtimes \mathrm{mod}\,A \arrow[r] \arrow[d, "\wr"] & \mathrm{mod}\,A \\
\mathrm{mod}\,A \otimes A \arrow[u]
\end{tikzcd}$$

$$\begin{tikzcd}
\mathrm{End}(\mathcal{F}) \arrow[r] & \mathrm{End}(\mathcal{F}) \otimes \mathrm{End}(\mathcal{F}) \arrow[d, shift right, "\cancel{\wr}"'] \\
& \mathrm{End}(\overline{\mathcal{F}}) \arrow[u, shift right]
\end{tikzcd}$$

$$\Delta(y^2) = \left((y \otimes 1 + 1 \otimes y) + \Sigma \right)^2$$

$$(y \otimes 1 + 1 \otimes y)^2 + 2(y \otimes 1 + 1 \otimes y)(\Sigma)$$

$$+ (\Sigma)^2$$

$$\left((x + x')^p - (x^p) - (x')^p \right)^2$$

$$(x + x')^{2p} - (x^p)(x+x')^p$$

$$- (x')^p (x + x')^p$$

$$- (x^p)(x + x')^p$$

$$- (x^{2p})$$

$$- (x^p)(x')^p$$

$$- (x')^p (x + x')^p$$

$$\begin{bmatrix} 1 & x & x^2 & x^3 & x^4 & y \\ & 1 & 2x & 3x^2 & 4x^3 & x^4 \\ & & 1 & 3x & 6x^2 & 2x^3 \\ & & & 1 & 4x & 2x^2 \\ & 0 & & & 1 & x \\ & & & & & 1 \end{bmatrix}$$

Δ

a_{1+}

$a_{0+1,3+1} = \binom{3}{0} x^3 =$

$a_{0+1,4+1} = \binom{4}{0} x^4$

$a_{1+1,3+1} = \binom{3}{1} x^2 = 3x^2$

$\frac{3!}{1!\,2!}$

$a_{2+1,3+1} = \binom{3}{2} x = 3x$

$\frac{3!}{2!\,1!}$

$a_{1+1,5+1} = \binom{5-1}{1} \frac{1}{5-1} x^{5-1}$

$= \frac{4!}{1!\cdot 4!} x^4$

$a_{3+1,3+1} = \binom{3}{3}$

$a_{2+1,5+1} = \binom{5-1}{2} \frac{1}{5-2} x^{5-2}$

$= \frac{4!}{2!\,2!} \frac{1}{3} x^3$

$2x^3$

$a_{3+1,5+1} = \binom{5-1}{3} \frac{1}{5-3} x^{5-3}$

$= \frac{4!}{3!\,1!} \cdot \frac{1}{2} x^2$

$= 2x^2$

$a_{4+1,5+1} = \binom{5-1}{4} \frac{1}{5-4} x^{5-4} = \frac{4!}{4!\cdot 1!} \frac{1}{1} x = x$

(8) $W(R) = \{(x,y) \mid x, y \in R\}$

$\cancel{(x+x')^3 - x^3 - (x')^3}$

$\sum_{i=0}^{p} \binom{p}{i} x^i y^{p-i}$

$\cancel{(x^2 + 2xx' + (x')^2)(x+x')}$

$\cancel{x^3 + x^2 x' +}$

$\frac{p!}{i!(p-i)!}$

$$(\varphi, \psi) : A_W \longrightarrow R$$
$$\overset{\shortparallel}{(x,y)}$$

$K[t, s]$

$\Delta(t) = t \otimes 1 + 1 \otimes t$

$\Delta(s) = s \otimes 1 + 1 \otimes s + \sum_{i=1}^{p-1} \binom{p-1}{i} t^i \otimes t^{p-i}$

$\varepsilon(t) = 0$ $\quad$ $\varepsilon(s) = 0$

$S(t) = -t$

$S(s) = -s$

$\cancel{t = (id, \varepsilon)\Delta(t)}$

$\sum_{i=1}^{p-1} \binom{p-1}{i}$

$s = (id, \varepsilon)\Delta(s)$

$= s + \varepsilon(s) + 0$

$0 = \varepsilon(t) = (S, id)\Delta(t)$

$S(t) = -t$

$\sum_{i=0}^{p-1} (-1)^i \binom{p-1}{i}$

$0 = (S, id)\Delta(s)$

$= S(s) + s + \sum_{i=1}^{p-1} \binom{p-1}{i} (-1)^i t^i \cdot t^{p-i}$

$= S(s) + s + \underbrace{\left(\sum_{i=1}^{p-1} (-1)^i \binom{p-1}{i}\right)}_{0} t^p$

$\sum_{i=0}^{p} (-1)^i \binom{p}{i} = (1 + (-1))^p = 0$

$S(s) = -s$

$\cancel{1} + \underbrace{\boxed{\sum_{i=1}^{p-1} (-1)^i \binom{p}{i}}}_{p} + \cancel{(-1)} = \underline{0}$
$ p$

$K[t,s]$

~~$C_0 = K$ $C_1 = \text{Span}\{1, t, s\}$~~

$C_0 = K \quad C_1 = \text{Span}\{1, s, t, t^2, \ldots\}$

~~$\Delta(t)$~~

$C_r = \text{Span}\{1, s, \ldots, s^r, t, t^2, \ldots\}$

$\Delta(t^2) =$

$\Delta(s^2) = (s\otimes 1 + 1\otimes s + \sum A_i t^i \otimes t^{n-i})(\qquad)$

~~$s\otimes$~~

$(s\otimes 1 + 1\otimes s)^2 +$

$\Delta(y)$

$\Delta(y^2)$

$K[x, 1/x]$

$\Delta(x) = x\otimes x$

$\Delta(1/x) = \frac{1}{x}\otimes\frac{1}{x}$

$C_0 = K$

$C_1 = \{1, x, 1/x\}$

$C_2 =$

$r = 3$

$\Delta($

$r=3 \quad C_3 = \{1, x, x^2, x^3\}$ $C_3 \otimes C_1 + C_1 \otimes C_0$

$C_2 = \{1, x, x^2\}$

$C_1 = \{1, x\}$

$\Delta(x^2) = (x\otimes 1 + 1\otimes x)^2$

$= (x^2\otimes 1 + 2(x\otimes x) + 1\otimes x^2)$

$C_3 \otimes C_0 + C_2 \otimes C_1 + C_1 \otimes C_2 + C_0 \otimes C_3$

i:

$A = K[x,y]$ $\quad V = \{1, y, x, ?\}$

$\Delta(x) = x \otimes 1 + 1 \otimes x$

$\Delta(y) = y \otimes 1 + 1 \otimes y + \sum_{i=1}^{p-1} \binom{p-1}{i} \frac{1}{p-i} x^i \otimes x^{p-i}$

for any $v_i \in V$ we need

$p(v_i) = \Delta(v_j) = \sum v_i \otimes a_{ij}$

since $y \in V$

$\Delta(y) =$ $\quad \Rightarrow \{x, \ldots, x^{p-1}\} \subseteq V$

$\Delta(x^k) = \sum_{i=0}^{k} \binom{k}{i} x^i \otimes x^{k-i}$ since $k \le p-1$, they're already in V

So $V = \{1, x, \ldots, x^{p-1}, y\}$

$\underset{j=1}{\Delta(1)} = \underset{1}{1} \otimes 1 \quad \Rightarrow a_{11} = 1$

$\underset{j=2}{\Delta(x)} = \underset{i=2}{x} \otimes 1 + \underset{i=1}{1} \otimes x \qquad a_{22} = 1$

$a_{12} = x$

$\underset{j=3}{\Delta(x^2)} = x^2 \otimes 1 + x \otimes 2x + 1 \otimes x^2 \qquad a_{33} = 1,\ a_{23} = 2x,\ a_{13} = x^2$

$\vdots$

$\Delta(x^k) = \sum_{i=0}^{k} \binom{k}{i} x^i \otimes x^{k-i} \qquad a_{i+1, k+1} = \binom{k}{i} x^{k-i}$

$\Delta(y) = y \otimes 1 + 1 \otimes y + \sum_{i=1}^{p-1} \binom{p-1}{i} \frac{1}{p-i} x^i \otimes x^{p-i}$

$a_{1, p+1} = y \quad a_{i+1, p+1} = \binom{p-1}{i} \frac{1}{p-i} x^{p-i}$

$a_{p+1, p+1} = 1$

b) ⇒ c) Suppose any irred. rep is one dim.

~~Let V be a rep of G w/ dim n, so $G \to GL_n$~~

Suppose $\rho : G \longrightarrow GL_n$ is an embedding of G. Then we have an n-dim rep V_1 associated to ρ.

Since $V_1 \subseteq V$ is an irred. rep in V it is one-dim by assumption.

~~If n=1, then $G \to GL_1$ is by def in $T_1 = GL_1$~~

~~Inducting on dimension,~~ since V_1 is

we claim that for such an embedding ~~V decomposes as a direct sum of one dim reps.~~ G acts by ~~triangul~~ ~~diagonal~~ upper triangular

~~Inducting on dimension~~

Since V_1 is one dim we have $V_1 = Kv_1$ for some v_1, so ~~G acts on~~ V/Kv_1 decomposes as a direct sum of irred. rep's, So there is a basis $[v_2], \dots, [v_n]$ s.t. $V/Kv_1 \cong \bigoplus_{i=2}^{n} K[v_i]$.

Now, we consider the basis $\{v_1, \dots, v_n\}$.

Since V_1 is an irreducible subrep. it is invariant. It suffices to show v_i is fixed by G a)=>c)

Suppose every linear rep has a onedim invariant subspace